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PROFESSIONAL RECOMMENDATION MEMORANDUM

Project Name: **Engineering Support Services and Recommendations for Roadside Safety Issues/Problems for Member States**

Sponsor: **Roadside Safety Pooled Fund**

Task 21-05: **Connecticut W-Beam Rail End Connection**

DATE: **August 04, 2021**

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Overview/Problem Statement

The technical memorandum presents the professional opinions regarding the structural sufficiency of the trailing end connections to provide adequate anchoring resistance as per the request of Connecticut Department of Transportation. The trailing end connection (Figure 1) located on the facet of concrete parapet exposed to traffic has been used to anchor the guard rails which were NCHRP Report 350 crash tested. It has been utilized on the downstream end of bridge parapets on limited access highways connecting for years without any in-service performance issues reported. With the implementation of the MASH testing requirements, the required test vehicles characteristics have changed resulting in greater impact loadings. Accordingly, Connecticut DOT is seeking to know if the connection is structurally adequate to provide appropriate anchoring resistance for their current and mash compliant guard rails being impacted downstream from this connection based on the new MASH testing requirements.

Figure 1. Trailing End Connection on Downstream End of Bridge Parapet

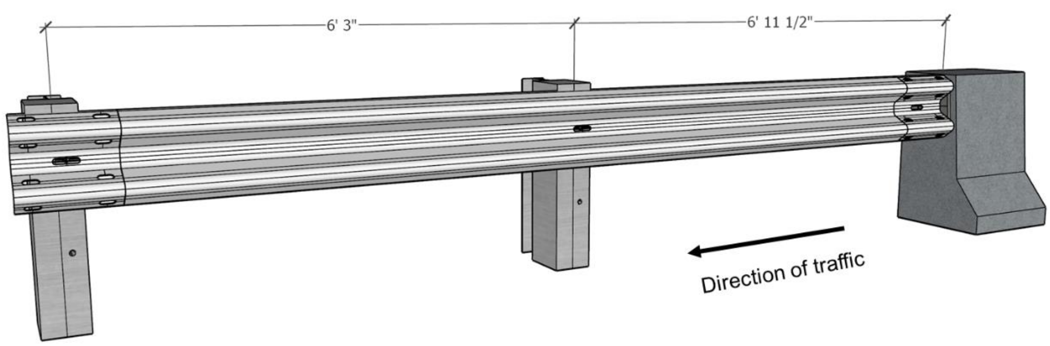
Analysis Approach

The overview of the trail end connection is shown in Figure 2. The connection is anchoring the W-beam rail with 3-anchor bolts to a concrete parapet. It is installed on divided limited access highways with only traffic passing the connection in the direction indicated in Figure 2(a). All the connected W-beam guard rail configurations are Test Level 3 barrier. To evaluate the trailing end connection for structural adequacy with respect to MASH impact loads, two loading cases are considered as shown in Figure 3.

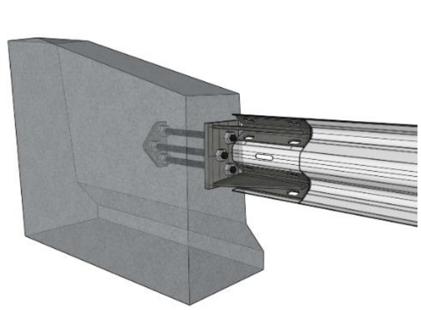
1. <u>Tension load</u> – A crash load downstream and away from the end connection may induce a large tension load to the end connection. The connection should possess a capacity stronger than the maximum tensile strength (113 kips) of the W-beam rail to prevent a premature connection failure take place before the rail beam develop its full strength.

2. <u>Shear load</u> – The vehicle could impact near the end of the guard rail (just past the end of the concrete barrier) and impart a high lateral force on the connection. The connection should be able to resist the applied shear load at the end of the span. The impact load for this shear case is assumed that 71 kips distributed over 4 feet from the end of the span as depicted in Figure 3(b).

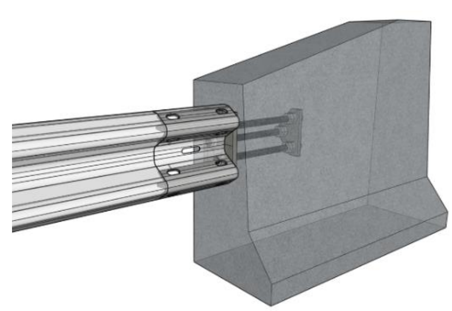
The end connection consists of two main parts – anchor bolts and steel bracket. Figure 4 shows some details of the end connection. The strength of the steel bracket is evaluated to check the steel strength of the connection plates and the weld strength. The anchor bolts are evaluated to check the steel strength of the bolts and concrete anchoring strength based on the ACI 318.



(a) Overview

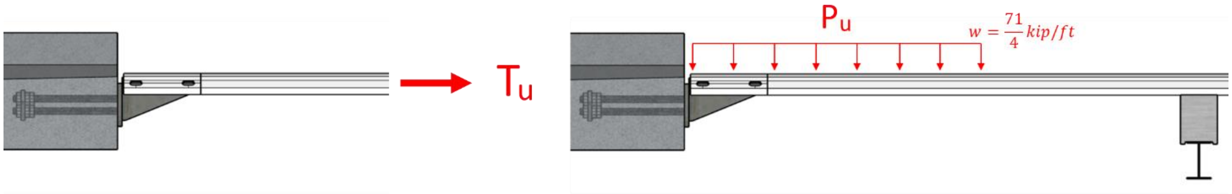


(b) Backside view of connection



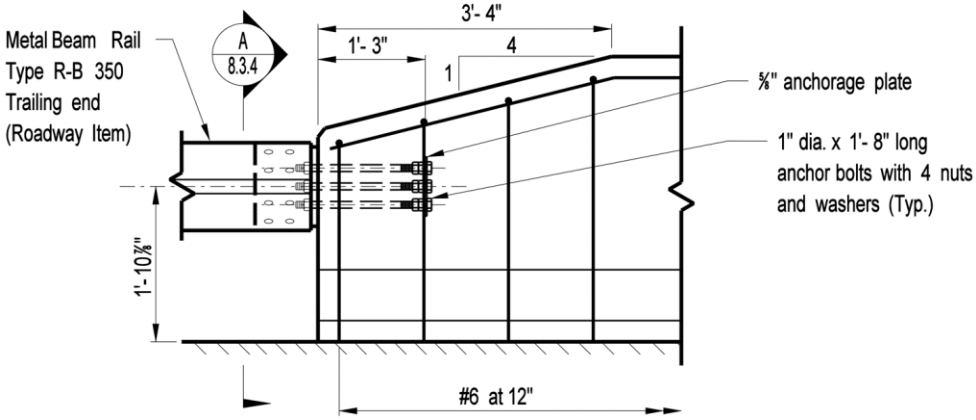
(c) Frontside view of connection

Figure 2. 3-D Rendering of Trailing End Connection

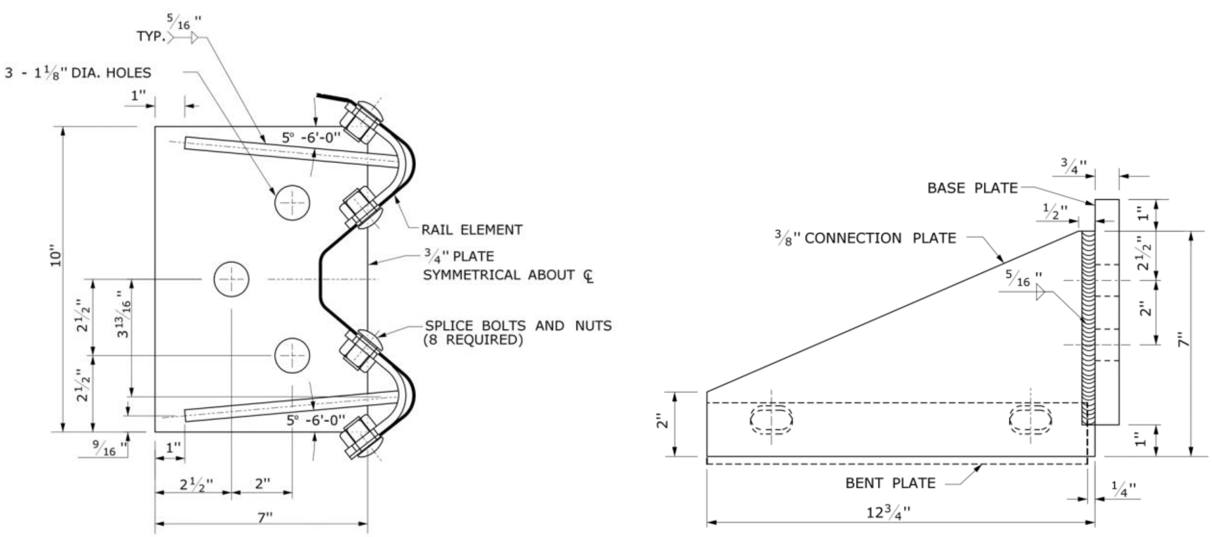


(a) Tension load *(b) Shear load*

Figure 3. Load Cases for Analysis



(a) Elevation of rail end



(b) Base plate and assembly *(c) Connection plate*

Figure 4. Details of Rail End Connection

ACI 318 categorized various failure modes of concrete anchors in tension and shear. Figure 5 shows each of the potential failure modes for the concrete anchors in tension and shear categorized by ACI 318. For a single or group of anchors subject to tensile loading, the anchors may subject to steel failure, pullout, concrete breakout, concrete splitting, side-face blowout, and bond failure. The anchor bolts used for the end connection are the cast-in bolts with an anchorage plate attached at the embedded side of the anchors. Therefore, the concrete splitting and bond failure modes are excluded from the evaluation. For the shear case, the anchor bolts are evaluated to check the steel failure, concrete pryout, and concrete breakout strength. Please refer to the attached worksheet for more details of the evaluation.

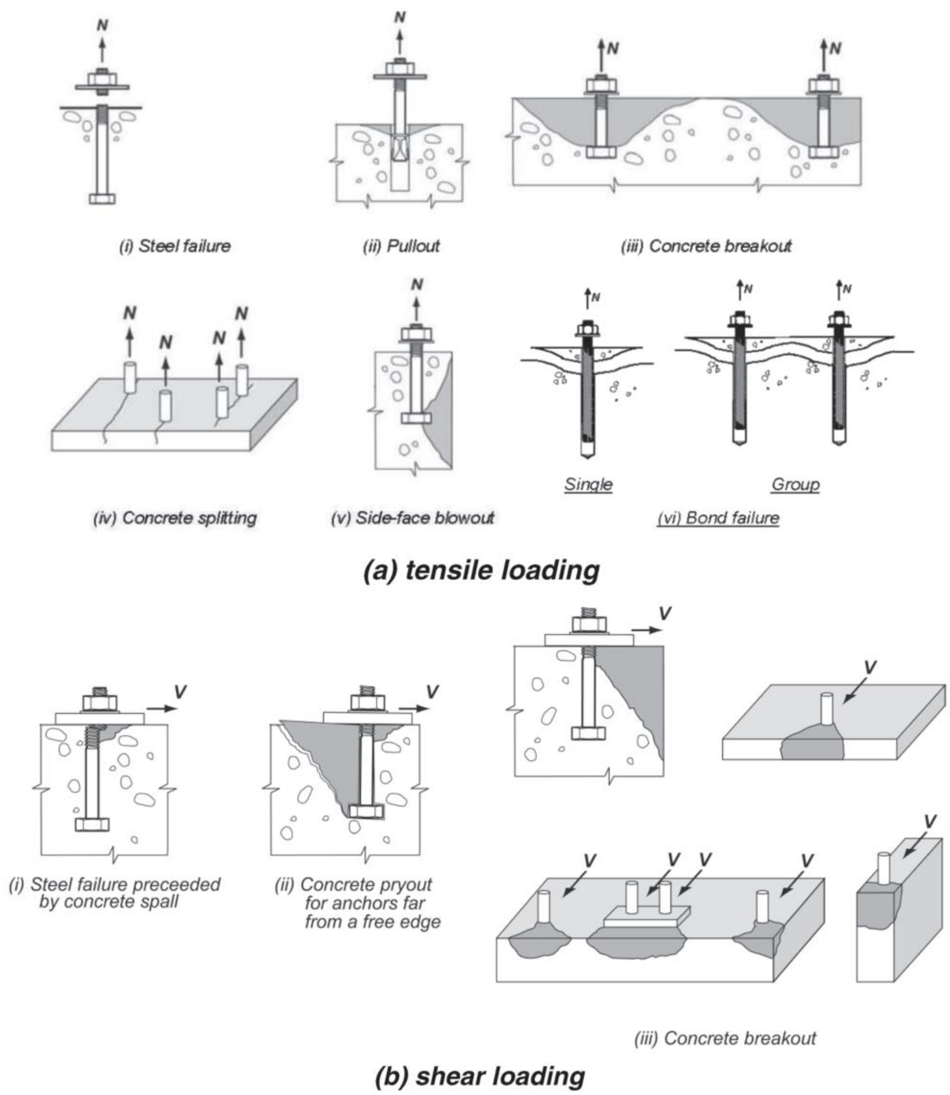


Figure taken from ACI 318

Figure 5. Failure Modes of Concrete Anchoring (ACI 318)

Analysis Result

The analysis results for the tension and shear load cases are listed in Table 1 and Table 2, respectively. The steel strength of the bolts and bracket as well as the weld strength has sufficient capacity to resist the demand. However, the concrete anchoring strength of the connection found to be inadequate to meet the demand for both tension and shear cases. The end connection is susceptible to concrete breakout failure.

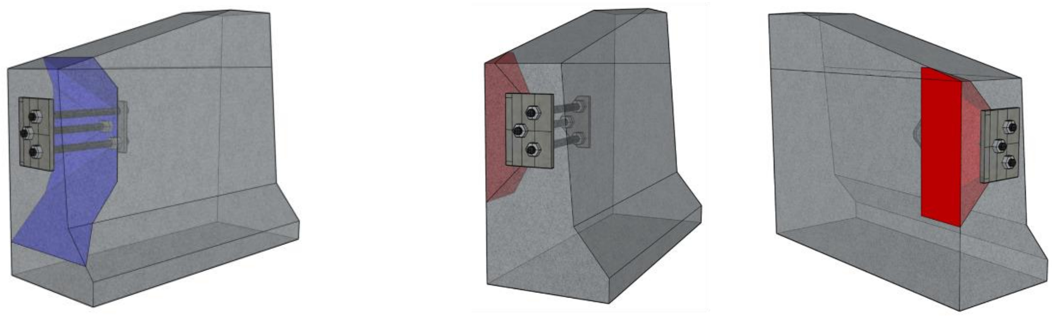
The code given equations yield a very conservative result for the concrete breakout strength because of the reduction factors due to the limited section dimension and load eccentricity. An alternative analysis method is therefore considered to calculate the concrete breakout strength. As shown in Figure 6, the failure plane for the concrete breakout failure is simulated by assuming the failure plane radiating out at a 45-degree from the end of the anchors at the embedded side. The area of the assumed failure plane is then worked out to calculate the concrete breakout strength by multiplying the shear strength of the concrete (assumed as $2\sqrt{f_c'}$). The alternative method gives a more realistic capacity estimations but still inadequate to meet the demand. It should be noted that, both the code equations and the alternative methods for the concrete breakout strength is solely based on the concrete strength. The presence of the reinforcement within the failure plane may improve the concrete breakout strength. Also, the compressive strength of the concrete is assumed as *3.6 ksi* in the calculation. Use of the actual strength of the concrete would give a more accurate estimation.

Table 1. Analysis Results for Tension Load

	Capacity (kips)	Demand (kips)	Check
Bolt strength:	53	48	O.K.
Concrete pullout:	1149	113	O.K.
Concrete breakout:			
- ACI 318	12	113	N.G.
- Actual failure area	60	113	N.G.
Side-face blowout:	248	113	O.K.
Connection plate:	96	56	O.K.
Weld strength:	37.6 (kip-ft)	37.5 (kip-ft)	O.K.

Table 2. Analysis Results for Shear Load

	Capacity (kips)	Demand (kips)	Check
Bolt strength:	38	17	O.K.
Concrete pryout:	120	51	O.K.
Concrete breakout:			
- ACI 318	16	51	N.G.
- Actual failure area	29	51	N.G.
Connection plate:	79	25	O.K.
Weld strength:	225	51	O.K.



(a) Assumed failure plane for tension load *(b) Assumed failure plane for shear load*

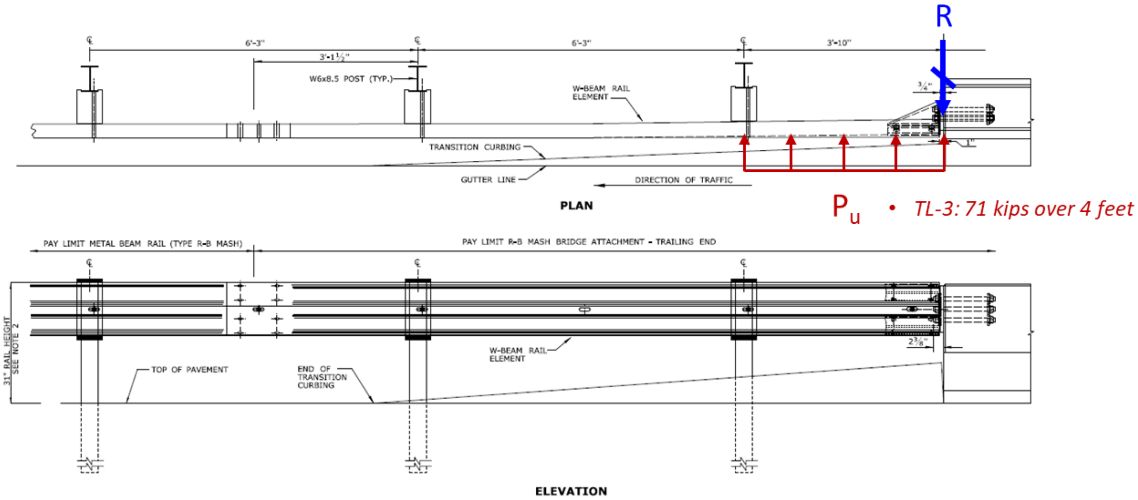
Figure 6. Simulated Failure Plane for Concrete Breakout Failure

Trails with Shorter Span Length

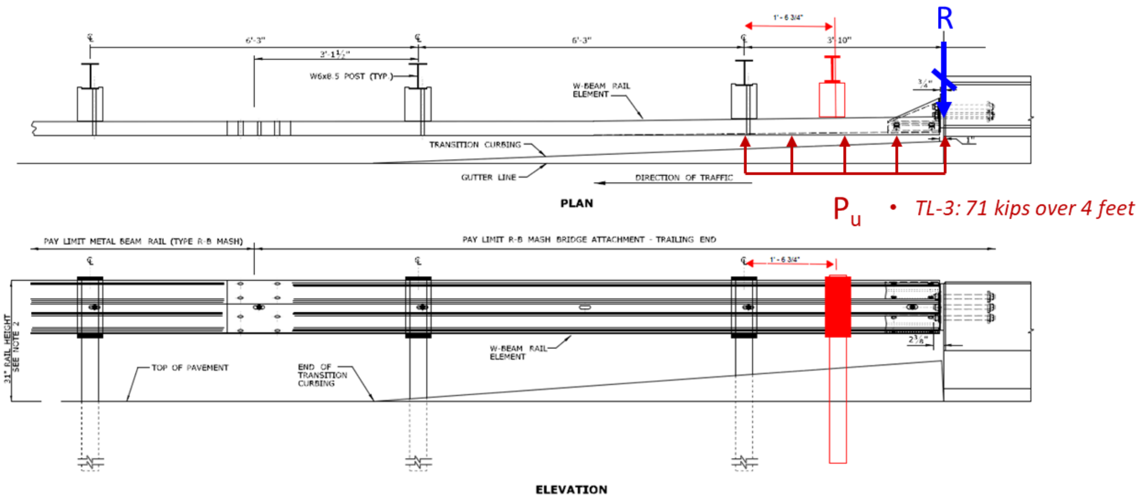
The analysis presented above is based on the configurations of the in-service trail. Another trail with a shorter span, as shown in Figure 7(a), is considered. The adjacent post is moved toward the trail end resulted a short span length of 3'-10" for the first span. The reaction on the trail end can be easily determined from static equations for such a case. However, the guardrail post is not a rigid and fixed support. The post with loadings will move and deflect. The amount of lateral force the post can resist is limited. Therefore, the majority of the lateral force will be resisted by the trail end. The analysis of the reaction force *R* on the trail end involves an indeterminate problem which depends upon the elastic properties of the post and the rail beams.

For a simplified evaluation, an assumption is made for the lateral force resistance of the guardrail post. It is conservatively assumed that the maximum lateral force the guardrail post can resist is 12 kips. If the post is a rigid support, the reaction force on the post will be the (nearly) half of the external force (35.5 kips) acting on the rail beam, which is definitely higher than the lateral force resistance of the guardrail post. By assuming the guardrail post is resisting up to 12 kips forces, it is saying that the rest of the external force (59 kips) is acting on the trail end connection. The capacities of the trail end are already specified in the previous section. Table 3 (Case A) compares the capacities with the new demands. A similar result is obtained. The trail end connection found to be vulnerable to concrete break out failure.

A second case, as shown in Figure 7(b), is also considered. A guardrail post is added in between the first span. The amount of the force on the end connection will be reduced as the additional support added in between the span. If the posts are rigid, the reaction forces will be 9 kips, 43 kips, and 18 kips for the end post, mid post, and the end connection. The forces on the posts, again, are higher than the assumed resistance. Similarly, the reaction force on the end connection for this case is 47 kips by assuming a single post can resist up to 12 kips. The capacities of the end connection are compared to the demand for this case in Table 3 (Case b). Though the reaction force on the end connection is reduced, the concrete break out strength of the end connection is still inadequate.



(a) Case A with 3'-10" span



(b) Case B with additional post in between the span

Figure 7. Trails with Reduced Span Length

Table 3. Analysis Results for Trails with Reduced Span Length

	Case A (3'-10" span length)			Case B (additional post in between span)		
	Capacity (kips)	Demand (kips)	Check	Capacity (kips)	Demand (kips)	Check
Bolt strength:	38	20	O.K.	38	16	O.K.
Concrete pryout:	120	59	O.K.	120	47	O.K.
Concrete breakout:						
- ACI 318	16	59	N.G.	16	47	N.G.
- Actual failure area	29	59	N.G.	29	47	N.G.
Connection plate:	79	30	O.K.	79	24	O.K.
Weld strength:	225	59	O.K.	225	47	O.K.

Summary and Recommendations

The structural adequacy of the rail end connection is evaluated against the new MASH TL-3 loading requirements. The steel strength, weld strength and concrete anchoring strength of the rail connection is evaluated for the potential failure modes. As the analysis results show, the end connection is found to be vulnerable to concrete breakout failure in both tension and shear load cases for MASH TL-3 impact loading conditions. Howevrer, the ACI code calculations are assumed to be very conservative. Considering the actual failure planes in tension, the calculations show this strength to be more reasonable but still deficient for MASH TL-3 impact conditions.

The trails with reduced span lengths are also analyzed. The reaction forces acting on the end conneciton for these cases are determined by assuming the guardrail post can resist a limited amount of lateral force (12 kips). Simillarly, the end conneciton was found to be vulnerable to concrete breakout failure. It is recommended that a more sophiscated anaylsis be performed (LS-DYNA finite element modeling simulations) to determine the performance of the guardrail end post connection. Full-scale crash testing can also be performed to determine the performance with to MASH Test Level 3.

In summary, the concrete breakout strength of the end connection was found to be inadequate to resist the impact condition of MASH TL-3. It is recommended that LS-DYNA simulations(s) or full-scale crash testing with respect to MASH TL-3 impact condition be performed to determine if this connection is adequate for MASH TL-3 impact conditions.

Reference

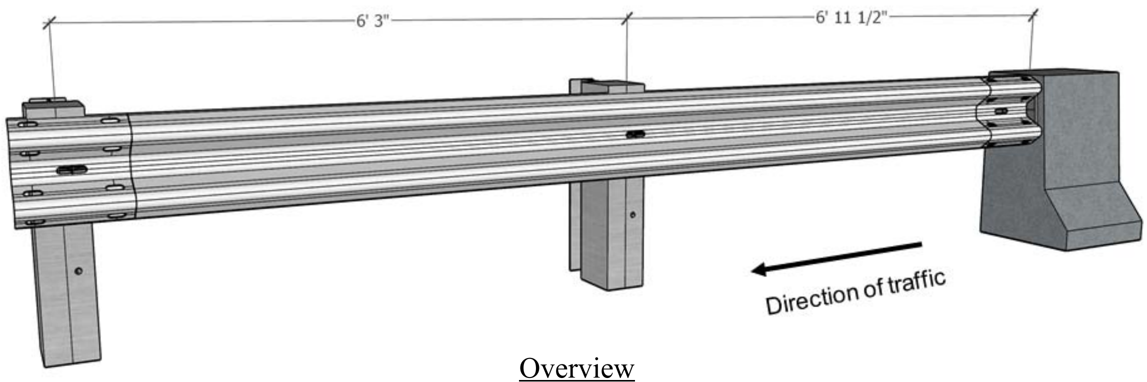
ACI 318-14 (2014). "Building code requirements for structural concrete and commentary." *2014 building code and commentary*, A. C. Institute, ed., Farmington Hills, MI : American Concrete Institute (ACI), Farmington Hills, MI.

Appendix.
Calculation Worksheet

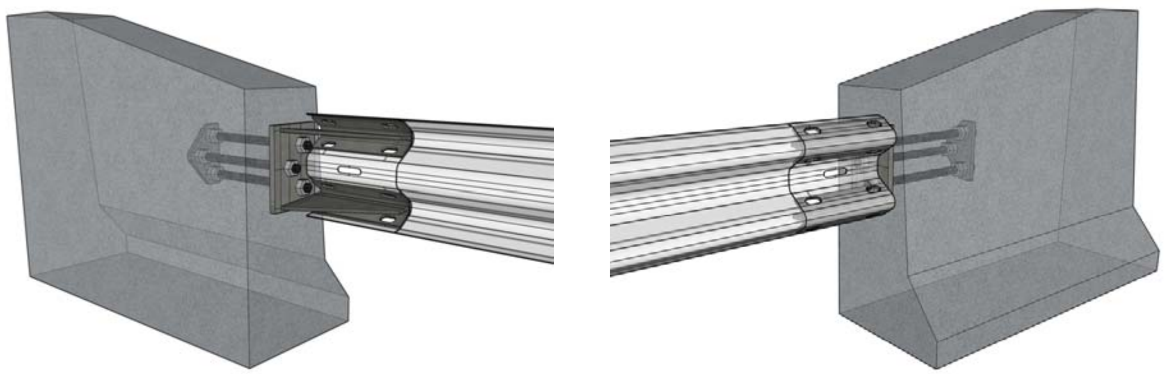
Texas A&M Transportation Institute

SUBJECT:
Professional Opinion 21-05
– W-Beam Rail End Anchor Connection

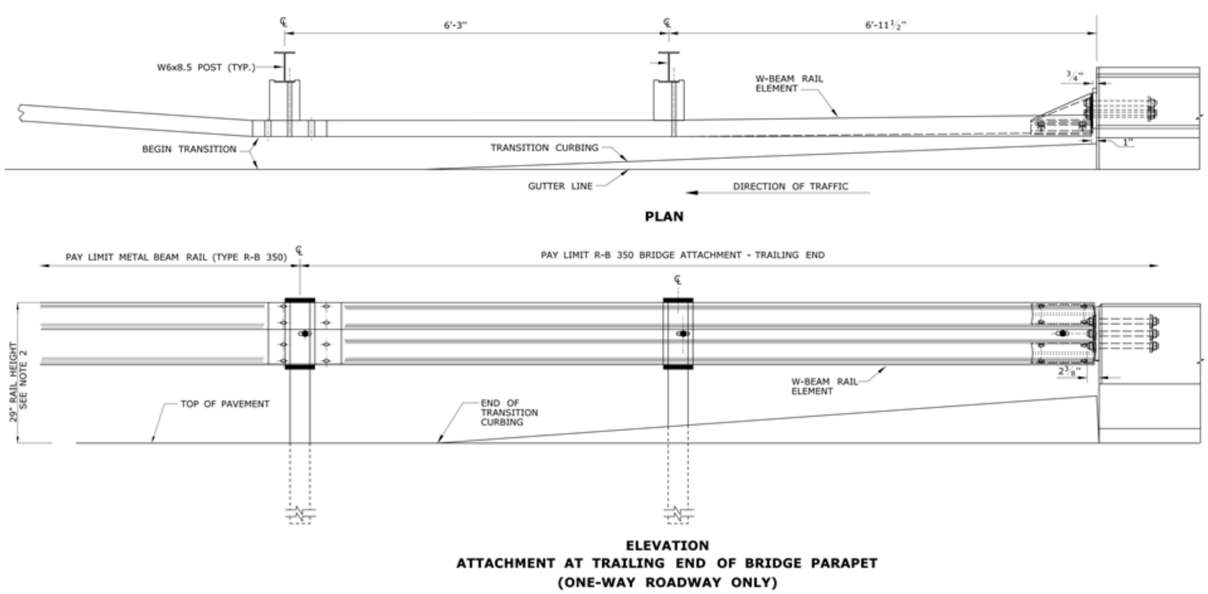
The sturcutral performance of an anchor conncetion at the end of a W-beam rail used to mount the guardrail to the concrete parapet is evaluted to check the compliance to the MASH requirement. The details of the W-beam rail as well as the anchored end connection are presented below.



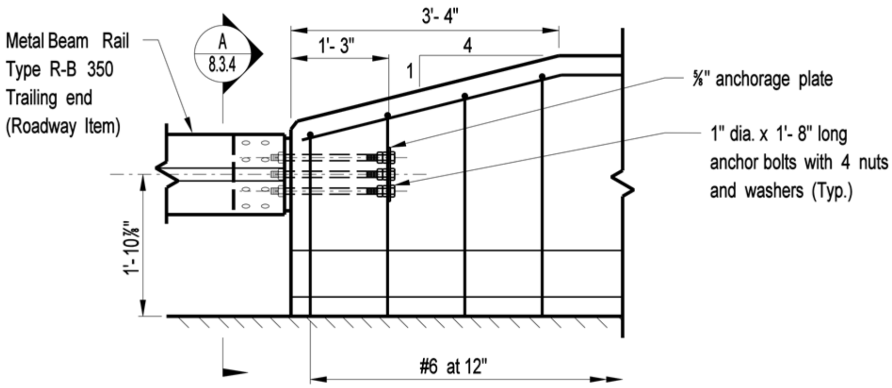
Overview



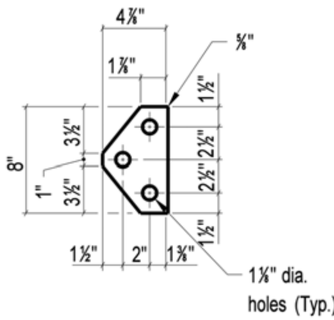
End Connection



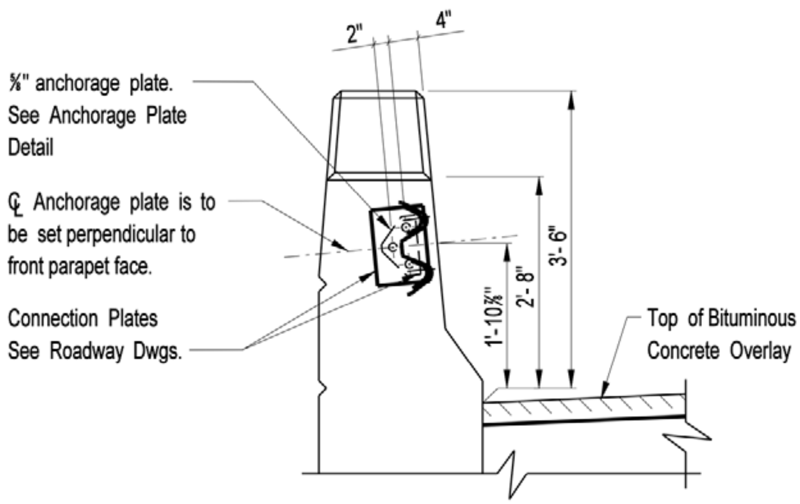
Schematic



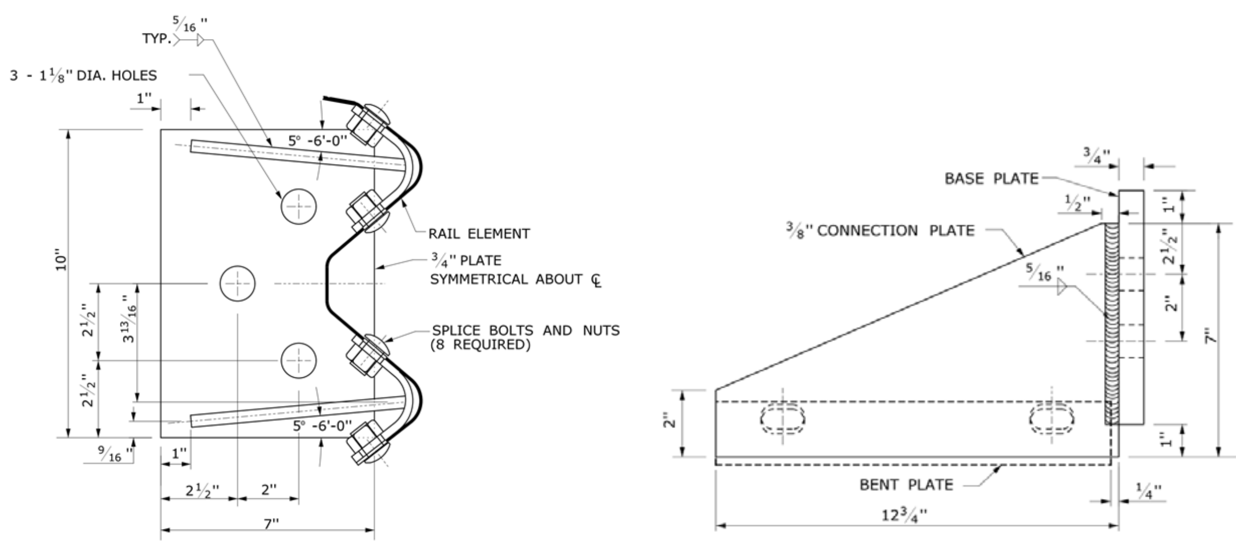
Rail End Elevation



Anchorage Plate



Rail End Attachment



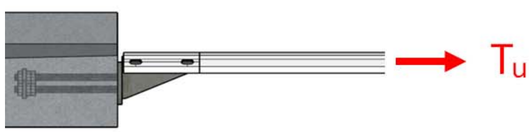
Connection Bracket

1. General Information and Input

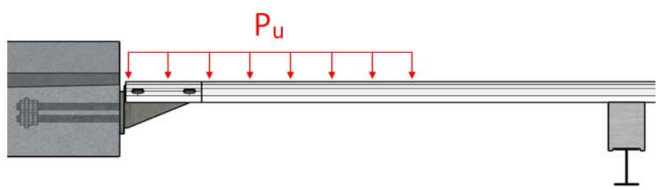
The rail end connection is evaluated for the following two load cases.

(1) **Tension load -** a crash load at the mid-region of the span may induce a tension load to the rail end. The end connection should poses a higher capacity than the maximum tensile strength of the W-beam rail to prevent a connection failure occurs before the rail beam develop its full strength.

(2) **Shear load -** the end connection also requires enough shear resistance when the crash load applied right at the end of the rail beam.



(1) Tension load



(2) Shear load

General properties of the structures are specified as follow.

W-beam (12-gauge):

$f_{y\_rail} := 50\ ksi$	*Yield strength*
$f_{u\_rail} := 70\ ksi$	*Tensile strength*
$A_{g\_rail} := 1.99\ in^2$	*Gross area*
$t_{rail} := 0.105\ in$	*Thickness*
$h_{slot} := \frac{29}{32}\ in$	*Depth of slotted holes for splice bolts*

Anchor bolt (A325):

$d_{bolt} := 1\ in$	*Diameter*
$f_{u\_bolt} := 90\ ksi$	*Tensile strength*
$h_{ef} := 15\ in$	*Effective embeddment depth*

End attachment:

$f'_c := 3.6\ ksi$	*Specified concrete sternght of parapet*
$E := 29000\ ksi$	*Young's modulus of steel plates and bracket*
$F_y := 50\ ksi$	*Yield strength of steel plates and bracket*
$F_u := 65\ ksi$	*Tensile strength of steel plates and bracket*
$F_{EXX} := 70\ ksi$	*Electrode classification number*

2. End Connection Subject to Tension Load

The strength of the anchor bolts, concrete anchorage, connection plate and the weld strength of the connection plate is evaluated against the maximum tensile strength of the W-beam rail.

2.1 Maximum tensile strength of W-beam

- Net area of W-beam,

$$A_{g\_rail} = 1.99\ in^2 \quad ; \quad t_{rail} = 0.11\ in \quad ; \quad h_{slot} = 0.91\ in$$

$$A_{n\_rail} := A_{g\_rail} - 4 \cdot t_{rail} \cdot h_{slot} = 1.61\ in^2$$

- Maximum tensile strength of W-beam,

$$f_{u\_rail} = 70\ ksi$$

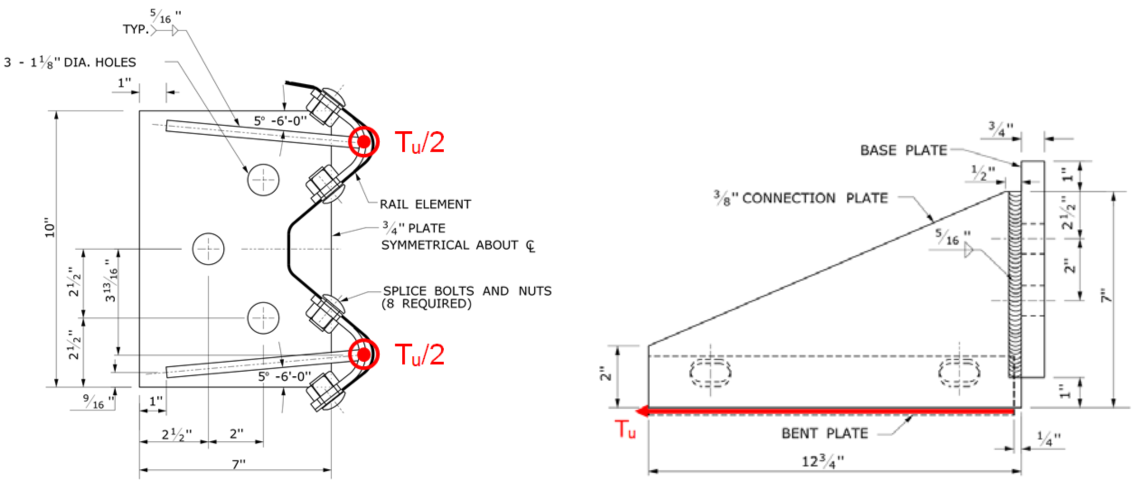
$$F_{nt\_rail} := f_{u\_rail} \cdot A_{n\_rail} = 112.66\ kip$$

- Tension load on end connection

$$T_u := F_{nt\_rail} = 112.66\ kip$$

2.2 Load eccentricity

The load eccentricty to the base plate, welding and the connection plate is calculated as follow.



** Tension load on bracket*

- Eccentricity to base plate

$$y_{base} := \frac{7 \cdot 10 \cdot \frac{7}{2} - \frac{\pi \cdot 1.125^2}{4} \cdot (2.5 + 2 \cdot 4.5)}{7 \cdot 10 - \frac{3 \cdot \pi \cdot 1.125^2}{4}} \cdot in = 3.49\ in$$

$$e_{base} := 7\ in + 1\ in - y_{base} = 4.51\ in$$

- Eccentricity to welding

$$e_{weld} := \frac{(7-1)}{2}\ \boldsymbol{in} + 1\ \boldsymbol{in} = 4\ \boldsymbol{in}$$

- Eccentricity to connection plate

$$e_{cp} := \frac{0.5 \cdot 7 \cdot \frac{7}{2} + 12.25 \cdot 5 \cdot 0.5 \cdot \frac{5}{3} + 2 \cdot 12.25 \cdot 1}{0.5 \cdot 7 + 12.25 \cdot 5 \cdot 0.5 + 2 \cdot 12.25}\ \boldsymbol{in} = 1.5\ \boldsymbol{in}$$

2.3 Bolt strength - tension

- Tension load on bolts

Assume the base plate is rotating along the bottom (left side edge in the above figure) axis, the tension load on each row of the bolt can be found as,

$$T_{ui} := \frac{M_u \cdot d_i}{\Sigma d_i^{\,2}} \quad \text{where} \quad M_u := T_u \cdot e_{base} = 508.62\ (\boldsymbol{kip} \cdot \boldsymbol{in}) \ ; \quad d_1 := 2.5\ \boldsymbol{in} \quad ; \quad d_2 := 4.5\ \boldsymbol{in}$$

$$T_{ub1} := \frac{M_u \cdot d_1}{d_1^{\,2} + d_2^{\,2}} = 47.98\ \boldsymbol{kip}$$ *Tension load on bottom (left) line bolt*

$$T_{ub2} := \frac{M_u \cdot d_2}{d_1^{\,2} + d_2^{\,2}} = 86.37\ \boldsymbol{kip}$$ *Tension load on second (right) line bolt*

$$T_{ub2\_single} := \frac{T_{ub2}}{2} = 43.19\ \boldsymbol{kip}$$ *Tension load on single bolt in second line*

- Tensile strength of bolts

$$A_{bolt} := \frac{\pi \cdot d_{bolt}^{\,2}}{4} = 0.785\ \boldsymbol{in}^2$$ *Nominal area of bolt*

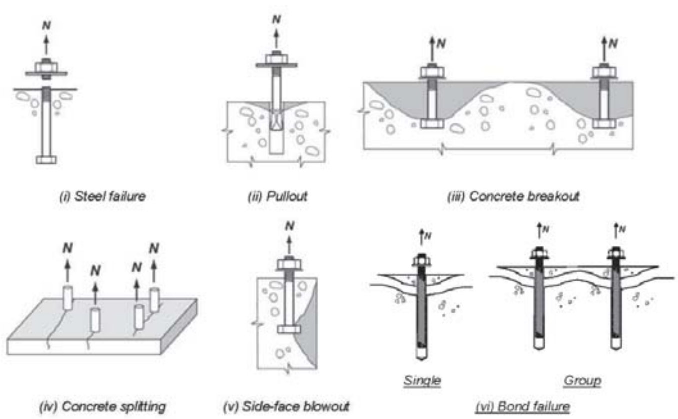
$$R_{n\_bolt} := f_{u\_bolt} \cdot (0.75 \cdot A_{bolt}) = 53.01\ \boldsymbol{kip}$$ *Nominal tensile strength*

$R_{n\_bolt} = 53.01\ \boldsymbol{kip}$ > Load on bottom line $T_{ub1} = 47.98\ \boldsymbol{kip}$ **(O.K.)**

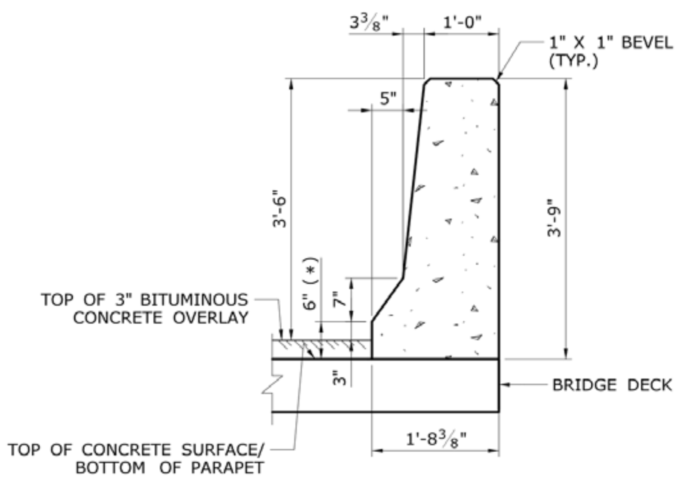
> Load on second line $T_{ub2\_single} = 43.19\ \boldsymbol{kip}$ **(O.K.)**

2.4 Concrete anchorage in tension

The concrete anchoring strength of the end connection is evaluated based on ACI 318 Ch. 17 - Anchoring to Concrete. For a single or group of anchors subject to tensile loading, the following failure modes are possibly took place.The anchor bolts used for the end connections are cast-in bolts with an anchorage plate embedded in the concrete. Also, a steel plate attachment is used. Therefore, the *concrete splitting* and *bond failure* are excluded from the evaluation. The anchors are evaluated for,



**Anchorage failure in tension - figure taken from ACI 318. Fig. R17.3.1*



**F-Shape Parapet - referred to Connecticut DOT database*

- **Pullout**
- **Concrete breakout**
- **Side-face blowout**

(**Steel failure is checked in the previous calcualtion*)

- Effective embeddment depth of anchors:

$h_{ef} = 15\ in$

- Maximum spacing of bolts along edge:

$s_{max} := 5\ in$

*Note, the dimension of the F-shape parapet shown in the figure is referred to CDOT database, which may differ from the actual dimensions.

- Top width of parapet:

$W_p := 12\ in$

- Edge distance of anchors:

$c_{a1} := 4\ in$	*to front face*
$c_{a2} := 6.625\ in$	*to top face*
$c_{a3} := 6\ in$	*to back side face*

- 2.4.1 Pullout strength in tension (ACI 318. 17.4.3)

$f'_c = 3.6\ ksi$	*Concrete sterngth*
$A_{brg} := 28.5\ in^2$	*Bearing area - area of the anchorage plate*
$N_p := 8 \cdot A_{brg} \cdot f'_c = 820.8\ kip$	*Pullout strength, ACI 318. Eq. 17.4.3.4*
$\psi_{c.p} := 1.4$	*Modification factor for concrete cracking, ACI 318. 17.4.3.6*
$N_{pn} := \psi_{c.p} \cdot N_p = 1149.12\ kip$	*Nominal pullout strength, ACI 318. Eq. 17.4.3.1*

$N_{pn}=1149.12\ kip$ > $T_u=112.66\ kip$ **(O.K.)**

The pullout failure defined here is the local cocnrete crushing around the anchor head at the embeded side of the anchor. For this case, the anchorage plate used at the embeded side resulted a higher bearing area, therefore, yiled a higher capacity.

- 2.4.2 Concrete breakout strength in tension (ACI 318 17.4.2)

Check for narrow section requirement, ACI 318. 17.4.2.3
(cannot fully develop failure plane in three sides of the section)

$1.5 \cdot h_{ef}=22.5\ in$ > $c_{a1}=4\ in$; $c_{a2}=6.63\ in$; $c_{a3}=6\ in$

A reduction to h_{ef} should apply.

h'_{ef} = larger of $\dfrac{c_{a2}}{1.5}=4.42\ in$ *Take maximum edge distance*

$\dfrac{s_{max}}{3}=1.67\ in$ *Take maximum bolt spacing*

$h'_{ef}:=\dfrac{c_{a2}}{1.5}=4.42\ in$ *Reduced effective embeddment depth for bolts*

Nominal concrete breakout strength in tension,

$N_{cbg}:=\dfrac{A_{Nc}}{A_{Nco}} \cdot \psi_{ec.N} \cdot \psi_{ed.N} \cdot \psi_{c.N} \cdot \psi_{cp.N} \cdot N_b$ *ACI 318. Eq. 17.4.2.1b*

$A_{Nco}:=9 \cdot {h'_{ef}}^2=175.56\ in^2$ *Projected failure area of single anchor w/ edge distance equale to or greater than 1.5h_{ef}, ACI 318. Eq. 17.4.2.1c*

$A_{Nc}:=249.45\ in^2$ *Projected failure area of actual anchor group*

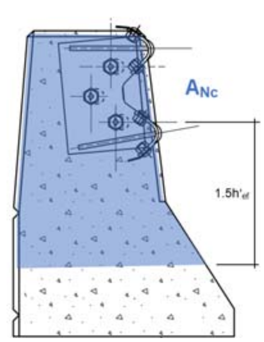
$N_b:=\left(k_c \cdot \lambda_a \cdot \sqrt{f'_c \cdot \dfrac{1000}{ksi}} \cdot {h'_{ef}}^{1.5} \cdot \dfrac{1}{in^{1.5}}\right)\ lbf$ *Basic concrete breakout strength of single anchor, ACI 318. Eq. 17.4.2.2a*

$k_c:=24$ *Coefficient for cast-in anchors*

$\lambda_a:=1$ *Coefficient for light weight concrete*

$N_b=13.37\ kip$

$e'_N:=e_{base}=4.51\ in$ *Load eccentricity to anchor group*



Projected area, A_{nc}

$\psi_{ec.N} := \dfrac{1}{\left(1 + \dfrac{2 \cdot e'_N}{3 \cdot h'_{ef}}\right)} = 0.59$ *Modification factor for load eccentricity, ACI 318. Eq. 17.4.2.4*

$\psi_{ed.N} := 0.7 + 0.3 \cdot \dfrac{c_{a1}}{1.5 \cdot h'_{ef}} = 0.88$ *Modification factor for edge effect, ACI 318. Eq. 17.4.2.5b*

$\psi_{c.N} := 1.25$ *Modification factor for concrete cracking, ACI 318. 17.4.2.6*

$\psi_{cp.N} := 1.0$ *Modification factor for spliting control, ACI 318. 17.4.2.7*

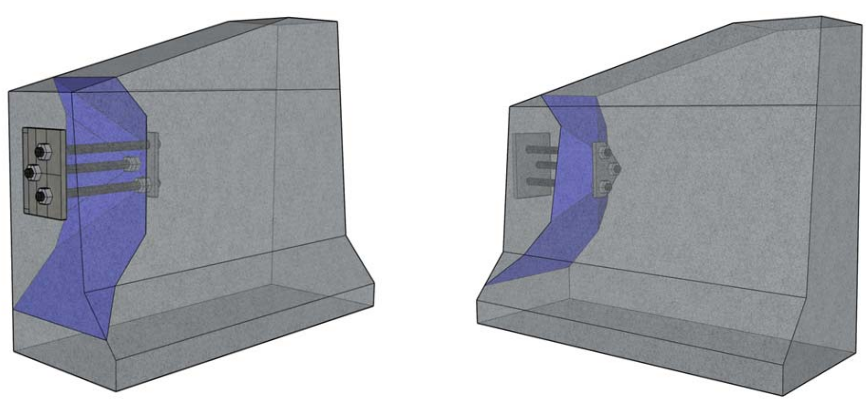
$N_{cbg} := \dfrac{A_{Nc}}{A_{Nco}} \cdot \psi_{ec.N} \cdot \psi_{ed.N} \cdot \psi_{c.N} \cdot \psi_{cp.N} \cdot N_b = 12.44\ \boldsymbol{kip}$ *ACI 318. Eq. 17.4.2.1b*

$N_{cbg} = 12.44\ \boldsymbol{kip}$ < $T_u = 112.66\ \boldsymbol{kip}$ (**N.G.**)

* It should be noted that, the concrete breakout strength in tension could be improved by considering the contribution of the concrete reinforecment within the region of the projected failure area. According to ACI 318. 17.4.2.9, *where anchor reinforcement is developed on both sides of the breakout surface, the design strength of the anchor reinforcement shall be permitted to be used instaed of concrete breakout strength.*

- 2.4.3 Concrete breakout strength in tension (actual area of failure plane)

Check the breakout strength based on the actual failure plane as shown. Assume the failure plane radiating out at a 45-degree. The area of the failure plane, shown in blue in the figures, is found to be 500 in^2.



$A_{ncb} := 500\ \boldsymbol{in}^2$

Projected failure plane for break out failure in tension

$N_{cb\_act} := 2 \cdot \sqrt{f'_c \cdot \dfrac{1000}{\boldsymbol{ksi}}} \cdot A_{ncb} \cdot \dfrac{\boldsymbol{lbf}}{\boldsymbol{in}^2} = 60\ \boldsymbol{kip}$

$N_{cb\_act} = 60\ kip$ < $T_u = 112.66\ kip$ (**N.G.**)

The estimated capacity for this case would be considered more realistic than the code estimation. It should be noted that, the breakout strength estimated here is purely based on the concrete strength. Any reinforecment exist within the failure plane would improve the concrete breakout strength of the connection.

- 2.4.4 Side-face blowout strength in tension (ACI 318. 17.4.4)

Nominal side-face blowout strength for single headed anchor with $h_{ef} > 2.5c_{a1}$,

$$N_{sb} := 160 \cdot c_{a1} \cdot \sqrt{A_{brg}} \cdot \lambda_a \cdot \sqrt{f'_c \cdot \frac{1000}{ksi}} \cdot \frac{1}{in^2} \cdot lbf = 205\ kip$$

ACI 318. Eq. 17.4.4.1

Nominal side-face blowout strength for multiple headed anchors with $h_{ef} > 2.5c_{a1}$, and $s_{max} < 6C_{a1}$,

$$N_{sbg} := \left(1 + \frac{s_{max}}{6 \cdot c_{a1}}\right) \cdot N_{sb} = 247.71\ kip$$

ACI 318. Eq. 17.4.4.2

$N_{sbg} = 247.71\ kip$ > $T_u = 112.66\ kip$ (**O.K.**)

2.5 Weld strength - tension

Reference: Table 5 - properties of Weld Treated as a Line, Design of Welded Structures, Omer W. Blodgett, 1982, pg. 7.4-7

$F_{EXX} = 70\ ksi$ *Weld strength*

$t_{weld} := \frac{5}{16}\ in$ *Weld thickness*

$d_{weld} := 6\ in$ *Length of weld*

$\theta_{weld} := 5.1\ °$ *Angle of loading*

$$S_w := \frac{d_{weld}^{\ 2}}{3} \cdot 0.707 \cdot t_{weld} \cdot 2 = 5.3\ in^3$$

from Table 5 - weld in two lines

$$M_{a\_weld} := S_w \cdot 0.6 \cdot F_{EXX} \cdot 2 \cdot \left(1 + 0.5 \cdot \sin\left(\theta_{weld}\right)^{1.5}\right) = 451.31\ kip \cdot in$$

Allowable weld strength, AISC Eq. J2-5

$$M_{u\_weld} := T_u \cdot e_{weld} = 450.63\ kip \cdot in$$

Moment apply on welding

$M_{a\_weld} = 451.31\ kip \cdot in$ > $M_{u\_weld} = 450.63\ kip \cdot in$ (**O.K.**)

2.6 Connection plate - tension

- Triangular plate subject to eccentric loading

$F_y = 50\ ksi$ — *Yield strength*

$t_{cp} := \frac{3}{8}\ in$ — *Plate thickness*

$h_l := 7\ in$ — *Lateral length of plate*

$\alpha := \text{atan}\left(\frac{12.25}{5}\right) = 67.8\ °$ — *Angle at bottom of plate*

- Tensile yielding — *AISC D2*

$$R_{n\_cp} := F_y \cdot t_{cp} \cdot \sin(\alpha)^2 \cdot \left(\sqrt{4 \cdot e_{cp}^{\ 2} + h_l^{\ 2}} - 2 \cdot e_{cp}\right) = 74.23\ kip$$

$R_{n\_cp} = 74.23\ kip$ > $0.5 \cdot T_u = 56.33\ kip$ **(O.K.)**

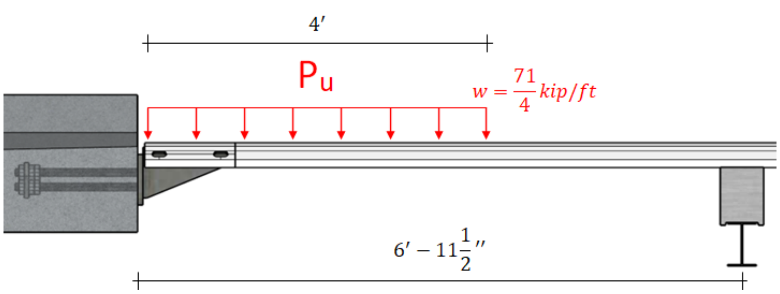
- Tensile rupture — *AISC D2*

$$R_{n\_cp} := F_u \cdot t_{cp} \cdot \sin(\alpha)^2 \cdot \left(\sqrt{4 \cdot e_{cp}^{\ 2} + h_l^{\ 2}} - 2 \cdot e_{cp}\right) = 96.51\ kip$$

$R_{n\_cp} = 96.51\ kip$ > $0.5 \cdot T_u = 56.33\ kip$ **(O.K.)**

3. End Connection Subject to Shear Load

For case of shear, place the impact load of 71 kips over 4 ft length from the end of the support.



$L_{span} := 6.96\ ft$

$l_{load} := 4\ ft$

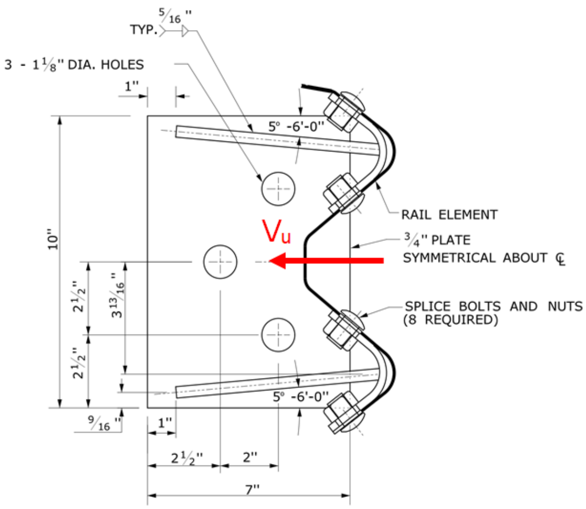
$w := 17.75\ \frac{kip}{ft}$

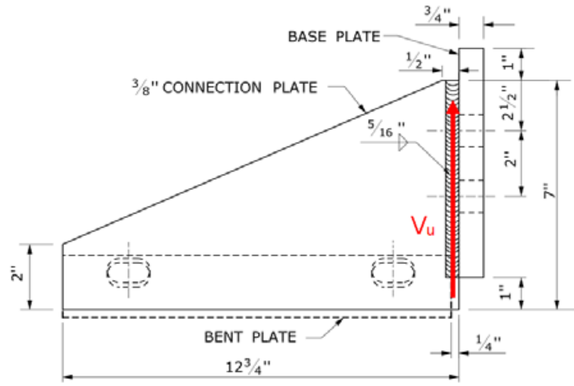
- Reaction at support

$$R_a := \frac{w \cdot l_{load} \cdot \left(L_{span} - l_{load} \cdot 0.5\right)}{L_{span}} = 50.6\ kip$$

Therefore, the total shear load on the end connection is,

$$V_u := R_a = 50.6\ kip$$





** Shear load on bracket*

3.1 Bolt strength - shear

$f_{nv\_bolt} := 48\ ksi$ — *Nominal shear stress of A325 bolts, when **threads are not excluded** from shear planes*

$$A_{bolt} := \frac{\pi \cdot d_{bolt}^{\ 2}}{4} = 0.785\ in^2$$

$R_{nv\_bolt} := f_{nv\_bolt} \cdot A_{bolt} = 37.7\ kip$ — *Nominal shear strength of bolt*

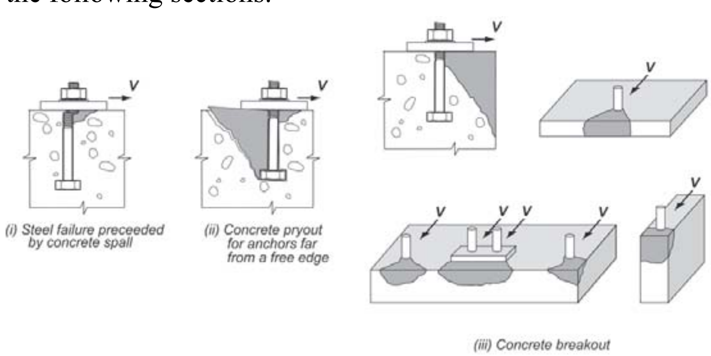
Assume the shear load is evenly distributing to the anchor bolts since a base plate is used.

$V_{u\_single} := \dfrac{V_u}{3} = 16.87\ kip$ — *Shear load on single bolt*

$R_{nv\_bolt} = 37.7\ kip$ > $V_{u\_single} := \dfrac{V_u}{3} = 16.87\ kip$ **(O.K.)**

3.2 Concrete anchorage in shear

The concrete anchoring strength for shear loading is evaluated for the following failure modes specified by ACI 318. Ch. 17. The steel strength of the anchors for shear is checked in the previous section. The concrete **pryout** strength and concrete **breakout** strength in case of shear is evaluated in the following sections.



**Anchorage failure in shear - figure taken from ACI 318. Fig. R17.3.1*

- Edge distance:

$c_{a1} := 8\ in$ — *To back side face*

$c_{a2} := 6.625\ in$ — *To top face*

3.2.1 Concrete pryout strength in shear (ACI 318. 17.5.3)

For cast-in anchors,

$N_{cpg} := N_{cbg} = 12.44\ kip$ — *Concrete breakout strength in tension*

$k_{cp} := 2.0$ — *For h_{ef} >= 2.5 in.*

Nominal pryout strength for group of anchors,

$V_{cpg} := k_{cp} \cdot N_{cb\_act} = 120\ kip$ — *ACI 318. Eq. 17.5.3.1b*

$V_{cpg} = 120\ kip$ < $V_u = 50.6\ kip$ **(O.K.)**

* Concrete pryout strength in shear is determined based on the concrete breakout strength in tension. The breakout strength that calculated based on the actual area of the failure plane, which gives more reasonable result, is used here to estimate the pryout strength.

3.2.2 Concrete breakout strength in shear (ACI 318. 17.5.2)

Nominal concrete breakout strength in shear for group of anchors,

$$V_{cbg} := \frac{A_{Vc}}{A_{Vco}} \cdot \psi_{ec.V} \cdot \psi_{ed.V} \cdot \psi_{c.V} \cdot \psi_{h.V} \cdot V_b$$

ACI 318. Eq. 17.5.2.1b

$A_{Vco} := 4.5 \cdot {c_{a1}}^2 = 288\ in^2$ — *Projected failure area of single anchor in a deep member w/ edge distance equal to or greater than 1.5c_{a1} in direction perpendicular to shear force, ACI 318. Eq. 17.5.2.1c*

$A_{Vc} := 304.4\ in^2$ — *Base area of truncated half-pyramid projected on side face of member where top of half-pyramid is given by axis of anchor row selected as critical*

$l_e := h_{ef} = 15\ in$ — *Load-bearing length of anchor in shear*

$$V_{b1} := \left(\left(8 \cdot \left(\frac{l_e}{d_{bolt}}\right)^{0.2} \cdot \sqrt{d_{bolt} \cdot \frac{1}{in}}\right) \cdot \lambda_a \cdot \sqrt{f'_c \cdot \frac{1000}{ksi}} \cdot (c_{a1})^{1.5} \cdot \frac{1}{in^{1.5}}\right) \cdot lbf = 18.67\ kip$$

ACI 318. Eq. 17.5.2.3

$$V_{b2} := \left(9 \cdot \lambda_a \cdot \sqrt{f'_c \cdot \frac{1000}{ksi}} \cdot (c_{a1})^{1.5} \cdot \frac{1}{in^{1.5}}\right) \cdot lbf = 12.22\ kip$$

ACI 318. Eq. 17.5.2.2b

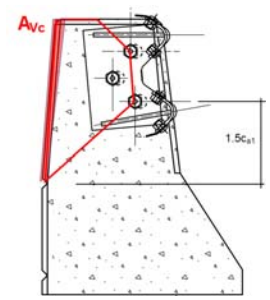
$V_b := min\left(V_{b1}, V_{b2}\right) = 12.22\ kip$ — *Basic concrete breakout strength of single cast-in headed anchor in shear, ACI 318. 17.5.2.3*

$\psi_{ec.V} := 1.0$ — *Modification factor for load eccentricity, ACI 318. Eq. 17.5.2.5*

$\psi_{ed.V} := 0.7 + 0.3 \cdot \frac{c_{a2}}{1.5 \cdot c_{a1}} = 0.87$ *Modification factor for edge effect, ACI 318. Eq. 17.5.2.6b*

$\psi_{c.V} := 1.4$ *Modification factor for concrete cracking, ACI 318. 17.5.2.7*

$\psi_{h.V} := 1.0$ *Modification factor for section depth, ACI 318. 17.5.2.8*



**Base area of truncated pyramid, Avc*

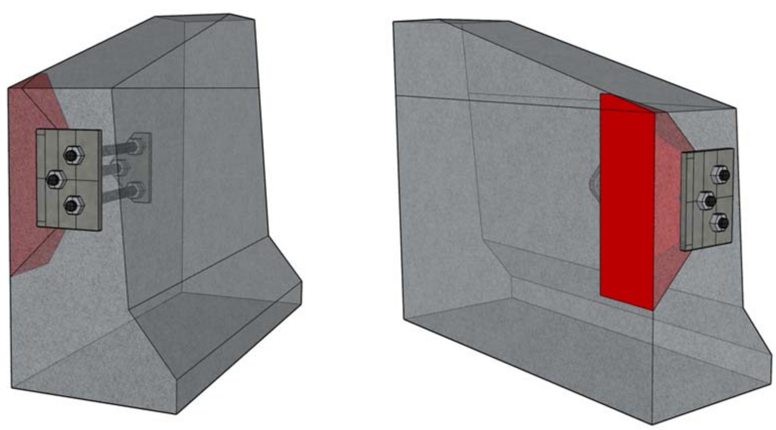
$$V_{cbg} := \frac{A_{Vc}}{A_{Vco}} \cdot \psi_{ec.V} \cdot \psi_{ed.V} \cdot \psi_{c.V} \cdot \psi_{h.V} \cdot V_b = 15.65\ kip$$ *ACI 318. Eq. 17.5.2.1b*

$V_{cbg} = 15.65\ kip$ < $V_u = 50.6\ kip$ (**N.G.**)

* Note that, the concrete breakout strength in shear is permitted to use the design strength of the anchor reinforcement instead (ACI 318. 17.5.2.9).

3.2.3 Concrete breakout strength in shear (actual area of failure plane)

Similarly, check the breakout strength in shear use the actual area of the failure plane.



$A_{vcb} := 240\ in^2$

**Projected failure plane for break out failure in shear*

$$V_{cb\_act} := 2 \cdot \sqrt{f'_c \cdot \frac{1000}{ksi}} \cdot A_{vcb} \cdot \frac{lbf}{in^2} = 28.8\ kip$$

$V_{cb\_act} = 28.8\ kip$ < $V_u = 50.6\ kip$ (**N.G.**)

Similar to the breakout strength in tension, the breakout strength in shear is also estimated based on the concrete strength. Any reinforcement engaged in the failure plane would improve the breakout strength in shear.

3.3 Weld strength - shear

$F_{EXX}=70\ ksi$ — *Weld strength*

$t_{weld}:=\frac{5}{16}\ in$ — *Weld thickness*

$d_{weld}:=6\ in$ — *Length of weld*

$\theta_{weld}:=5.1\ °$ — *Angle of loading*

$$R_{nv\_w}:=0.6 \cdot F_{EXX} \cdot 0.707 \cdot t_{weld} \cdot d_{weld} \cdot 4 \cdot \left(1+0.5 \cdot \sin\left(\theta_{weld}\right)^{1.5}\right)=225.66\ kip$$

Allowable weld strength, AISC Eq. J2-5

$R_{nv\_w}=225.66\ kip$ > $V_u=50.6\ kip$ **(O.K.)**

3.4 Connection plate - shear

$F_y=50\ ksi$ — *Yield strength*

$E=29000\ ksi$ — *Young's moduls*

$t_{cp}:=\frac{3}{8}\ in$ — *Plate thickness*

$h_{cp}:=7\ in$ — *Depth of connection plate*

$A_w:=t_{cp} \cdot h_{cp}=2.63\ in^2$

$\phi_v:=1.0$; $C_v:=1.0$ *for* $\frac{h_{cp}}{t_{cp}}=18.67 < 2.24 \cdot \sqrt{\frac{E}{F_y}}=53.95$ — *AISC G2.1*

$R_{nv\_cp}:=0.6 \cdot F_y \cdot A_w \cdot C_v=78.75\ kip$ — *AISC Eq. G2-1*

$R_{nv\_cp}=78.75\ kip$ > $0.5 \cdot V_u=25.3\ kip$ **(O.K.)**

4. Result Summary

4.1 End connection subject to tension load

⅋ Bolt strength:	$R_{n\_bolt}=53.01\ kip$	>	$T_{ub1}=47.98\ kip$	(**O.K.**)
			$T_{ub2\_single}=43.19\ kip$	(**O.K.**)
⅋ Concrete pullout strength:	$N_{pn}=1149.12\ kip$	>	$T_u=112.66\ kip$	(**O.K.**)
⅋ Concrete breakout strength:				
- ACI 318	$N_{cbg}=12.44\ kip$	<	$T_u=112.66\ kip$	(**N.G.**)
- Actual failure area	$N_{cb\_act}=60\ kip$	<	$T_u=112.66\ kip$	(**N.G.**)
⅋ Side-face blowout strength:	$N_{sbg}=247.71\ kip$	>	$T_u=112.66\ kip$	(**O.K.**)
⅋ Weld strength:	$M_{a\_weld}=37.61\ kip \cdot ft$	>	$M_{u\_weld}=37.55\ kip \cdot ft$	(**O.K.**)
⅋ Connection plate:	$R_{n\_cp}=96.51\ kip$	>	$0.5 \cdot T_u=56.33\ kip$	(**O.K.**)

4.2 End connection subject to shear load

⅋ Bolt strength:	$R_{nv\_bolt}=37.7\ kip$	>	$V_{u\_single}=16.87\ kip$	(**O.K.**)
⅋ Concrete pryout strength:	$V_{cpg}=120\ kip$	<	$V_u=50.6\ kip$	(**O.K.**)
⅋ Concrete breakout strength:				
- ACI 318	$V_{cbg}=15.65\ kip$	<	$V_u=50.6\ kip$	(**N.G.**)
- Actual failure area	$V_{cb\_act}=28.8\ kip$	<	$V_u=50.6\ kip$	(**N.G.**)
⅋ Weld strength:	$R_{nv\_w}=225.66\ kip$	>	$V_u=50.6\ kip$	(**O.K.**)
⅋ Connection plate:	$R_{nv\_cp}=78.75\ kip$	>	$0.5 \cdot V_u=25.3\ kip$	(**O.K.**)

4.3 Conclusion

The end connection is found to be susceptible for the *concrete breakout failure* for both tension and shear load cases. The code given equations yield a very conservative result. Use of the actual area of the failure plane is giving a more reasonable strength estimation, but still insufficient to meet the demand. Therefore, based on the corete breakout strength of the actual failure area (28.8 kips), this connection is likely acceptable for strength for MASH TL-2 (27 kips over 4 feet) impact conditions of 19.2 kips.